Game Theory from an Engineering Perspective

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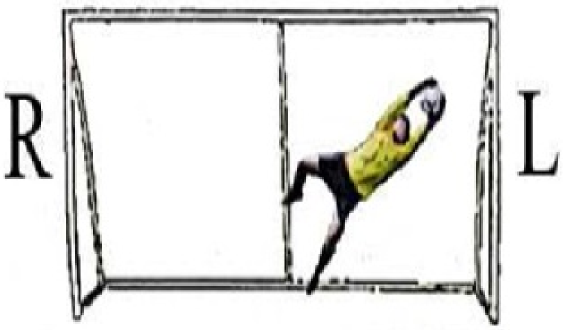
A much common misunderstanding regarding Game Theory comes from the different connotations the word 'game' has. An immediate connect that 'game' makes, is to the structured play undertaken for enjoyment, and which is also sometimes used as an education tool. In recent times, the word 'game' brings to the mind, a thriving industry, which is in computer games, many of the repertoire being based on the art and science of game theory. One prominent example is the game of Hex, invented by John Nash, the Nobel prize winning Game theorist. Though Nash suggested, the optimal size for this board game as 14x14, so far computer scientists have been able to solve only up to a size of 9x9. The challenge is still open!

1 Introduction

All the expressions or meanings given in connection with 'game' conjures up images of several nuances of human interactions. Some of the interactions perceived are comparison, competition, cooperation, conflict, collaboration, collusion, perhaps even conversation and other forms of communication. Most of these interactions, when examined from the perspective of derivation of benefit, are strategic in nature. Game Theory is said to mathematically model strategic interactions. In order to establish the connection between the games we play and strategical interaction, some typical examples are examined next.
In any field game, without doubt, 'competition' between the teams is the 'rule of the game'. Without cooperation between the team members and coordination of different positional players, no team can hope to be at the top of the game. In recent times, collaborating with manufacturers of sporting equipments and top-notch industries have become game changers. Unfortunately, collusion of players and umpires, who are fair game to the lure of the lucre, with bookies and their agents have rendered the sporting field, not all fun and games. However, in all the above interactions, a set of strategies in the custody of all interacting participants (players in the game theory context,) is at play. Obviously, each strategy is examined by the 'players' and accepted or discarded based on the maximization of benefits that can be derived, as an outcome.
A brief outline of what Game Theory is in order.

1.1 Preliminaries

A simple sub-game is used for illustrating the constituents concepts of Game Theory. Here, we have an example of a penalty shoot-out shown in the figure given below. It is what is explained as a 'zero-sum game' in the Game Theory parlance.



GK PT	L	R
L	0.8	1
R	1	0.5

Figure 1: Game Theory in a Soccer game

It is called as a zero sum game because 'someone's gain is someone's loss'. The theory part lies in the following analysis. There are only two players taking part in the interactions. Here, player 1 is the shooter who is taking the penalty turn. Player 2 is the goal keeper. Both the players have a set of action choices, but they are inter-dependent. Some of these are listed below, with a view to continue with the topic rather than expose the limited vision of the author on the beautiful game of football.

- If the shooter aims left while taking the kick, swerve left.
- If a right side kick is anticipated, guard the goal post accordingly
- If a central position is targeted by the shooter don't swerve
- If the kick is going to be out of the bounds of the post, stay put

Let us look at the strategies the goalkeeper may employ.

- If the shooter is A (B/C/D), swerve left (right/ centre / stay put)
- This is called a pure strategy and bases on information. It is deterministic.
- If the shooter is A, swerve left half the time and right half the time, probabilistically
- A similar strategy is employed by the goalkeeper for all players.
- This is mixed strategy

Now we come to the outcome of the interaction, based on the strategy used. In Game Theory, the benefit gained is quantified as payoff. The payoff table in the figure is interpreted as follows.

- It is the shooter who is counting his blessings.
- When both the players swerve to opposite sides, the shooter stands to gain (if the shot is accurate!). Hence the point or score marked against his account is one.
- If both parties swerve to the same side, say left, the score will be influenced by the quality of the kick and the capacity of the goalkeeper for 'save'; marked 0.8 here
- If the kick and swerve are both to the right, the shooter is shown to have a lesser edge (0.5)

1.2 The game of chess

Board games are big benefactors of GT, especially chess. This is another game based fully on strategic moves. It is played by two players who play 'back' and 'white'. There are several moves that are available to players at different stages of the game. A few moves that the black player considers is also shown with the moves available to the white player and the resulting outcomes.

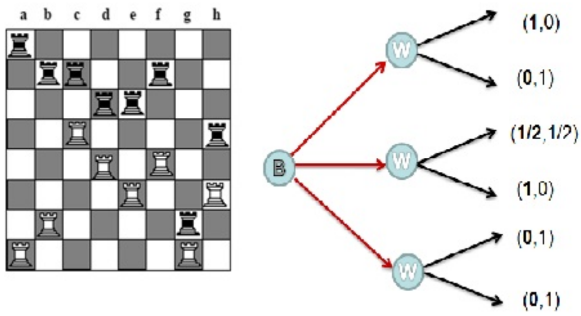


Figure 2: The game of chess

1.3 The games people play

Game theory is the general theory of strategic behaviour. Economic situations are treated games. But what is the importance of strategic interactions? And why model it mathematically? And Researchers have classified interactions into five: feelings, verbal ad non-verbal communication, behaviour and personalty. Quantification of these fields are through three dimensions, affecting all the five fields:

1. positive, negative
2. weak - strong
3. Passive - active

Human interactions come under various branches of study, mostly in Psychology and specifically in Cognitive Science. According to the subject that deals with Game Theory, there are several definitions to it. Some sweeping comments are given below:

- Game Theory is the mathematics of interactions
- If interactions are repeated, then we have Cooperative Game Theory
- A study to mathematically determine best strategy under given conditions to optimize the outcome
- Quantification of a rational human's preferences over some objects for maximizing utility
- Abstraction of real complex situations
- From mathematical models of competitive situations, finding acceptable if not optimal strategies in conflict situations

Obviously, the inter-disciplinary nature inherent in the definitions is indicative of several engineering applications.

2 Game Theory from an Engineering perspective

Before some applications are discussed it may do well to have an overview on why game theory is a good tool to model engineering problems. One issue is that process flows, production technologies, machine operations and control, communication technologies, internet, construction management, transportation, cryptography etc. are all engineering of interactions. There are also situations of conflict or competition in many engineering problems. Sometimes optimization becomes a strategic decision. In all the above cases, game theory can be used to model the proble. Such a modelling of the game, representing the conlict sutuation, requires the following constituents.

- Elements in a game (model for the conflict situation)
- Players, Choices/moves, Outcomes, Payoffs, Rules of the game
- Players/agents/participants in the game- humans, groups, organizations, machines, technologies or techniques, etc.
- A Strategy is a plan of action for a player whereby she has a decision rule for choosing a set of moves at every situation in the game
- Strategies are pure or mixed with probabilistic randomization
- A game based on perfect or imperfect information of the moves

2.1 Modelling and Solution

For engineering problems, players are mostly technological solutions. It may do well to remember that Game Theory is not exactly an optimization problem model, constrained or otherwise. Another check point while deciding to use the model is to remember that in Game Theory the variables are live. The solutions are dependent upon some factors given below.
Roughly there are two types of models: Competitive or non-co-operative game model and its anti-thesis, the Cooperative Game model. Competitive GT/ Non-cooperative GT models situations when simultaneous moves are made or when coalitions that decide or alter their strategies are not formed. An example is a Zero-sum game (where one's gain is another's loss). On the other hand Cooperative GT is applicable when moves are sequential and when benefits of co-operation are such that 'the whole is bigger than the parts'.
Thus the solutions emerge via several methods or approaches.

- Desired Outcome based Approach
- Based on players, characteristic function
- Payoffs or sharing of benefits based approach
- Based on attitude of players, sharing ideas

Though not explicitly visible, Mathematics including set theory has a major role in arriving at the solutions, be it co-operative or non-cooperative game theoretic model which is used.

2.2 Mathematics for solutions

It comes as no surprise that the solution techniques of the game theoretic models are the contributions of great Mathematicians. Of the 10 or more Nobel prizes awarded to Economics, since inception, are for Game theory based work and almost all contributions were made by

Mathematicians.
In brief, the ideas involved in the solution techniques associated with Non-cooperative Game Theory are given below. Professor John Nash, who recently passed away of 'A beautiful mind" fame, was the first proponent of Non-cooperative Game Theory and perhaps the first Nobel Prize winner too.

- Dominated strategy is a strategy that yields a payoff for one of the players that is less than or equal to the payoff for some other strategy for all actions of the opponent.
- Iterated Elimination of Dominated Strategies
- Most famous solution: Min-max solution: John Nash: Nash Equilibrium
- Mixed Strategy: A strategy from several possibilities using a probability distribution

In Cooperative Game Theory, as advocated by Professor Robert Aumann, there are again several options to obtain a solution space.

- Backward induction is a technique to solve a game of perfect information
- Most famous solution: Shapley Value
- Determining the core of the solution
- Socially Structured Transferable utility
- Marginal solution is most popular

Quite a lot of research is happening around these areas based on application areas.
In the social scenario, linguistic communication, organizational welfare, public health, local and global environmental change, river water sharing, resource management, etc. Replenishing resource management dilemma has come to be a major engineering problem. A brief overview of some engineering applications of Game Theory follows.

3 Engineering Applications of Game Theory

Quite a lot of research is still focusing on applications of game theoretical applications in engineering arena. Hence what is given here is by no means comprehensive.

3.1 Game theory and Civil engineering

Three major areas in civil engineering where research can be seen with game theory (GT) as a support structure are transportation, structural engineering and construction management.
Assuming the premise: Managing construction projects involves some conflicts among the players who are the Stakeholders, main contractors , among main and sub-contractors. A game has an objective or aim of achieving a win-win situation for every player. Yet, players' decisions lead to the worse and critical conditions for all involved in the construction projects. And it happens when conflicts arise while executing the project. This is because parties in the project act on behalf of themselves; in GT benefits of whole parties are considered. Let us have a brief look at construction mamanegement application.

1. Game Theory can recognize and clarify the behaviours of parties involved in the project to construction project problems. This is the modelling of the problem
2. GT can describe how interactions of different parties (players) -stakeholders (client), main contractors or subcontractors can lead to project evolving – Outcomes of the problem

3. GT and optimization methods results differ. Since in the latter, parties act to get best results for the whole system; in GT each party tend to act in which can lead to the most logical outcome for him; not necessarily the best result for the whole system

4. Solution method suggested is the use of Nash quilibrium.

3.2 Computer Science and Engineering

A very short but evocative list is appended below without much explanation, since it is the computer scientists who have made use of game theory.

1. Internet
2. Routing: routers and high speed links
3. Job scheduling
4. Competition in client-server systems
5. Peer-to-peer systems
6. Cryptology
7. Network security
8. Sensor networks
9. Game programming

3.3 Communication Engineering

Other similar areas of application with special research interest are

- Access Pricing of Wireless Mesh Networks
- Interaction of ISPs: Distributed Resource Allocation and Revenue Maximization
- Interaction of Multiple Overlay Routings
- Interaction between Overlay Routing and Underlay Routing
- Power control algorithm based on game theory in cognitive radio system
- Intrusion Detection Systems
- Power control problem in a CDMA-like s/m
- GT applied to network reliability

3.4 Electrical Engineering

The most prominent application area is in the recent electricity markets. In the current power scenario, game theory concepts are exploited in the complex Transmission Network Expansion Planning (TNEP) problem, all commerce associated with generation, transmission and distribution, operational strategies etc..
Game theory finds use in distributed control, Reactive Control for Transmission Overload Relief Based on Sensitivity Analysis and Cooperative Game Theory etc.
This is not an exhaustive collection of engineering applications. Management sciences have

made use of GT in several areas including for production. But it is time to conclude that we have touched the tip of the iceberg.

4 Conclusion

An interesting dilemma that we all face is recaptured here. When there is a drought, a person may think as follows: No matter what other community members do, watering the garden is better because it is unnecessary for one to exercise restraint if all others are restraining as well. However, if none restrain, it is futile for one to do so as one has no big impact on the whole water supply

The paradox : If the entire community reasons this way, water supply will dry up fully but if all cooperate and exercise restraint (acts irrationally) water supply will be spared.

In terms of Game Theory, Immanuel's Kant's categorical imperative : "Choose only a strategy which, if you could will it to be chosen by all the players, would yield a better outcome from your point of view than any other". This then, becomes a moral solution to the Prisoner's Dilemma i.e. only a cooperative choice can result. This is because the personal choice of defecting, if made universal, is contradictory to one's personal interest.

Perhaps the apt concluding line would be Kant's fundamental principle of morality: "Act only on such a maxim through which you can at the same time will that it should become a universal law."